EFFECT OF WITHDRAWAL FLOW VELOCITY ON THE COMPOSITION OF A TWO PHASE SYSTEM IN A MIXING TANK

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When a two phase mixture containing solids dispersed in a liquid medium is continuously fed and withdrawn from a mixing tank then, at steady state, the concentration of solids in the tank will equal the inlet and outlet concentrations only at the conditions of isokinetic withdrawal where the velocity in the withdrawal tube is the same as the approach velocity at the tip of the discharge opening. For a preferred withdrawal position the data is analysed to give an empirical correlation from which the steady state concentration of solids in the tank can be calculated if the withdrawal velocity is known for glass beads–water system stirred in a tank using turbine type mixing impeller.

Frequently it is assumed that to maintain a given concentration of a finely divided catalyst in a continuous flow reactor with its two phase contents well mixed by a rotating turbine, it is only necessary to feed a steady flow at the desired concentration of the solids in the carrier fluid. Implicit in it is that, at steady state the exit stream concentration of the catalyst will be same as in the feed and the concentration in the tank will also be the same. This, however, can only occur under carefully controlled conditions of isokinetic withdrawal, where the approach and withdrawal velocities at the tip of the discharge opening are equal. For non-isokinetic withdrawal the concentration of the solids in the tank, at steady state, could be more or less depending upon the velocity of withdrawal, diameter of the discharge opening, position of the mixing impeller with respect to the opening, density difference between the solid and liquid phases and the size of the solids.

Sharma[1], Rushton[2] and Řeháková and Novosad[3,4] have reported experimental confirmation of the above observation. The present work on glass beads–water system is directed to study the quantitative effects of the parameters involved and to propose empirical relations, if the data permit, for an otherwise theoretically complicated phenomenon.

THEORETICAL

The radial velocity at a horizontal distance r from the center of a turbine stirring the contents of a cylindrical vessel is given by the relation[2]

$$u_r = BND^2/r \,. \qquad (1)$$

If a withdrawal tube is positioned in line with the flow direction at the same r and if an withdrawal or exit velocity, u_e, is imposed in the withdrawal tube then, three cases as illustrated in Fig. 1 can occur, depending upon the magnitude of u_e.

In case *b* when the withdrawal velocity u_e is equal to the approach velocity u_r then the two phase stream enters undiverted and non-modified in terms of its solid contents. Consequently the concentrations of solids in the withdrawn stream remain unchanged *i.e.* if C_W is the concentration in the withdrawn stream and C_T is the concentration in the approaching stream then, $C_W = C_T$ for $u_r = u_e$. This situation is termed as isokinetic withdrawal and the exit velocity at the tip of the discharge opening could be named as isokinetic velocity, u_i. For the two non-isokinetic cases, the more inertial solid particles change their direction rather slowly with the result that the withdrawn stream is depleted of solids (*a*) where $C_W < C_T$ and $u_e > u_i$ or becomes richer in solids (*c*), where $C_W > C_T$ and $u_e < u_i$.

Řeháková and Novosad[3,4] have extended this basic concept to propose a model of two phase withdrawal, the success of which is yet to be experimentally confirmed.

EXPERIMENTAL

A typical configuration of the experimental setup is given in Fig. 2. The cylindrical tank was equipped with four standard baffles so adjusted that the withdrawal opening was midway between two of them. Six flat-blade stainless steel turbines were used. Discharge opening was put in line with the axis of the turbine. Table I shows the values of the geometric variables used for glass beads–water system in the present study.

A weighed amount of glass beads was put into the tank. Water was added upto a height equal to the diameter of the tank. Turbine was positioned from the tank bottom at a distance equal to 1/3rd of tank diameter and was placed in line with the discharge opening. The rotational speed of the turbine was kept always above the value calculated from the relation given by Zwietering[5] to the extent that homogeneity of solids concentration throughout the tank volume could be assumed. After letting the contents of the tank being stirred for a few minutes, the discharge valve in the outlet tube was opened and a sample of 100 ml of the mixed phase was collected in a centrifuge tube and the time, t_{100}, noted for its collection. The contents of the tube were later analyzed for concentration as apparent volume per cent which was related to the concentration in actual weight per cent through a linear calibration graph. Keeping all the conditions of the run the same the velocity of withdrawal was changed and the value of withdrawn concentration, C_W, was noted for the new value of t_{100}. The data covered at least a range from about 20 per cent below to 20 per cent above the isokinetic velocity. It should be noted that for a given area of discharge opening

$$u_e = 100/(t_{100} \cdot A) \qquad (2)$$

and

$$u_i = 100/(t_i \cdot A) \,, \qquad (3)$$

where t_i is the isokinetic time to collect 100 ml of the mixed phase *i.e.* at $t_i = t_{100}$, $C_W = C_T$.

RESULTS AND DISCUSSIONS

A few typical plots of C_W *versus* t_{100} are shown in Fig. 3, where evidently, a linear correlation exists between them. There is also a consistency in the observation that at isokinetic withdrawal ($u_e = u_i$), the withdrawn concentration tended to be the same as the concentration originally put in the tank. This indicates that the solids

TABLE I
Geometric Specifications of the Measured Systems

Particle size, μ	T, m	D, m	H, m	D_e, m . 10^3	Z, m . 10^3
60	0·305	0·102	0·305	2·4; 3·2; 4·8	0; 12·7; 38·1
250	0·457	0·152	0·457	2·4; 3·2; 4·8	0; 12·7; 38·1

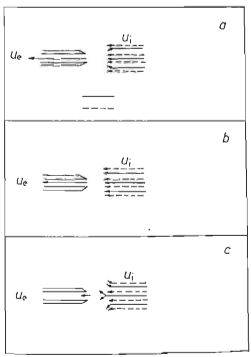

FIG. 1
Flow Behaviour Near Withdrawal Tube
——— Liquid flow lines, – – – – solid partial flow. *a* $u < u_e$, $C_W < C_T$; *b* isokinetic withdrawal $U = u_e$; $C_W = C_T$; *c* $u > u_E$, $C_W > C_T$.

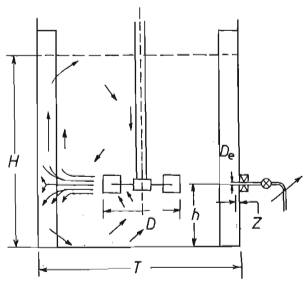

FIG. 2
Tank Configuration with Variable Head to Give Different u_e

concentration in the approaching stream can be taken equal to the average concentration, C_T, in the tank. Observed and from Eq. (*1*) calculated times t_{100} for isokinetic, withdrawal fall on a straight line with a slope of 45° for all the runs comprising

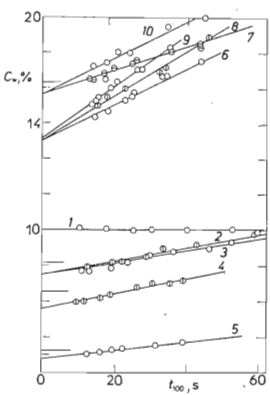

FIG. 3

Effect of Withdrawal Rate on Withdrawn Concentration

1 Water in kerosene; *2* Run No 13, 545 rpm; *3* Run No 12 440 rpm; *4* Run No 20 450 rpm; *5* Run No 26, 453 rpm. *6* Run No 36, 450 rpm; *7* Run No 54, 450 rpm; *8* Run No 37, 500 rpm; *9* Run No 41, 490 rpm; *10* Run No 53, 500 rpm.

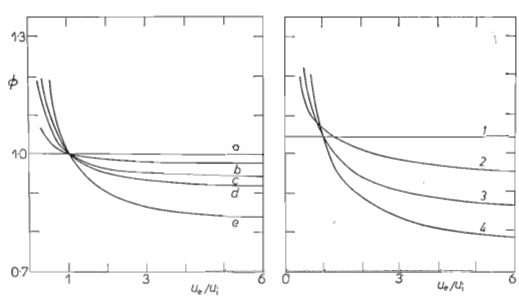

FIG. 4

Separation Coefficient *vs* Velocity Ratio, u_e/u

A: *a* Kerosene–water; *b* glass–water, 60μ, $Z/D_e = 4{\cdot}0$; *c* glass–water, 250 μ, $Z/D_e = 2{\cdot}667$; *d* glass–water, 250 μ, $Z/D_e = 4{\cdot}0$; *e* glass–water, 250 μ, $Z/D_e = 15{\cdot}96$. B: Reference[4]. *1* Polystyrene–water, 750 μ; *2* plexiglass–kerosene, 550 μ; *3* kerosene–sugar, 605 μ; *4* glass–water, 180—900 μ.

of different combinations of geometric variables. This confirms the validity of Eq. (*1*) and also establishes that the value of u_i can directly be obtained from basic plots of C_W *versus* t_{100}, which implies that solids concentration in the approaching stream is the same as average solids concentration in the tank.

Řeháková and Novosad[4] report value of separation coefficient C_W/C_T expressed as Φ equal to 1·04 instead of unity at isokinetic withdrawal conditions. According to them, there are no indications of an error in calculating u_i, on the other hand, it is mentioned that the average concentration in the tank may not be the same as the concentration in the vicinity of the discharge opening. It should be noted that contrary to the findings of the present study they obtained same Φ-curves for a wide range of particle diameters. Fig. 4*B* represents the curves of Řeháková and Novosad while 4*A* represents the curves obtained from present experiments as derived from basic $C_W - t_{100}$ plots. It can be observed that the basic character of the two sets of curves closely resembles giving indications that Φ *versus* $(u_e/u_i)^{-1}$ might results in a linear relationship.

For further analysis the basic $C_W - t_{100}$ curves would be used. The straight line relationship with the condition that $C_W = C_T$ at $u_e = u_i$ yields

$$\frac{C_W - C_T}{C_T} = \frac{m t_i}{C_T}\left(\frac{t_{100}}{t_i} - 1\right), \tag{4}$$

where m is the slope of $C_W - t_{100}$ line.

Since, $t_{100}/t_. = (u_e/u_i)^{-1}$

$$(\Phi - 1) = K\left[\left(\frac{u_e}{u_i}\right)^{-1} - 1\right], \tag{5}$$

where $K = m t_i/C_T$.

Fig. 5 shows the data plotted in the format of Eq. (*5*). The slope, K, it can be observed is a function of the ratio Z/D_e and the particle diameter. With increasing values of Z/D_e or particle diameter, the slope increases. Another important trend can be observed that, K is independent of m, C_T and t_i. This is because the curevs corresponding to different C_T and t_i fall on a common line. Remembering that, $K = m\,t_i/C_T$, it calls for establishing the ratio of $m\,t_i/C_T$ as constant for various values of C_T, t_i and m used in the present work. A scrutiny of Table II reveals that indeed for different experimental runs, this ratio, within experimental errors, remains constant, changing only with the value of Z/D_e or the particle size. This reinforces the earlier observation more comprehensively.

A plot between K *versus* Z/D_e is shown in Fig. 6. The values of K represent the arithmetic mean for a specified Z/D_e as listed in Table II. From the figure, following correlation results

$$K = 0{\cdot}045(Z/D_e)^{0{\cdot}5} . \qquad (6)$$

It should be noted that above equation is valid for insertion length of withdrawal greater than $T/36$. For smaller values (data is not taken) right to the case where discharge opening is flush with the wall (data is taken), stagnation region occurs and separation effects will follow a different course.

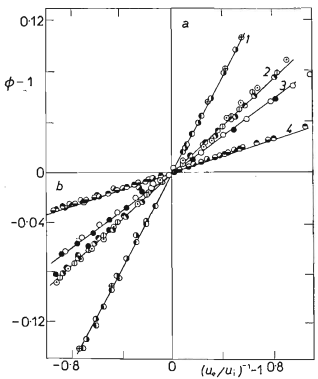

FIG. 5

Φ *vs* u_e/u_i Plot in the Format of Eq. (5)

Point	Z, cm	D_e, cm	C_T, %	μ
● ○	1·27	0·476	16·3	250
⊙ ⓘ	1·27	0·317	8·15	250
◓ ◒	1·27	0·317	16·3	60
◐ ◑	3·81	0·239	16·3	250
◴ ◵	1·27	0·317	16·3	250
◕	1·27	0·317	8·15	60
⊕	3·81	0·239	8·25	250

a u_e/u_i 1, *b* u_e/u_i 1; 1 $Z/D_e = 15{\cdot}96$; 2 $Z/D_e = 40$; 3 $Z/D_e = 2{\cdot}667$; 4 $Z/D_e = 4{\cdot}0$, 60μ.

Incorporating Eq. (6) in Eq. (5) finally, for 250 μ glass beads–water system

$$(\Phi - 1) = 0{\cdot}045(Z/D_e)^{0{\cdot}5}\left[\left(\frac{u_e}{u_i}\right)^{-1} - 1\right]. \qquad (7)$$

The constant will change for different bead sizes. This is indicated as the K value read from Table II for 60μ particle size does not fall on the 250μ-curve for a specified Z/D_e value of 4·0. More experiments are, therefore, needed to establish the relationship between the separation coefficient and the particle size. As an illustration: if the withdrawal velocity is one third of the isokinetic velocity then according to Eq. (7) the separation coefficient will be 1·18 showing that the concentration in the outlet will be 18 per cent greater than in the tank.

K, naturally, is expected to be a function of physical properties of the system including the particle size also. The present study has limitations since the data is obtained for the glass beads–water system only. The data of Řeháková and Novosad, however, could be used since their work pertains to different systems in the same geometry.

Because of the basic similarly in the nature of $\Phi - (u_e/u_i)$ curves, the data of these authors could be analysed in a similar way as done in the present study. Thus, plots between $(\Phi - 1)$ and $[(u_e/u_i)^{-1} - 1]$ were drawn and the K-values for different systems were noted. Fig. 7 shows the variation of K with the dimensionless density difference of the systems used. A straight line passing through origin suggests that

$$K = \text{Const.}\frac{\Delta\varrho}{\varrho_1}, \qquad (8)$$

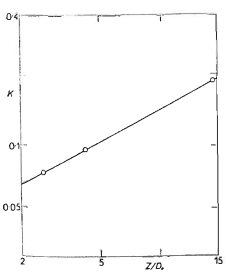

FIG. 6
Dependence of K on Z/D_e, 250 μ

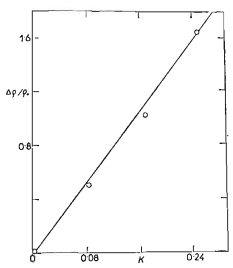

FIG. 7
Dependence of $\Delta\varrho/\varrho_1$ on K

where $\Delta\varrho$ is the density difference between the two phases *i.e.* $(\varrho_s - \varrho_1)$ and ϱ_1 is the density of the liquid. It should be noted that according to the data of Řeháková and Novosad, the constant in Eq. (*8*) should be independent of particle size contrary to the findings of the present experiments. The form of Eq. (*8*) with unspecified constant, however, could be assumed tentatively. In that case, the final equation would be

$$(\Phi - 1) = C\left(\frac{\Delta\varrho}{\varrho_1}\right)\left(\frac{Z}{D_e}\right)^{0.5}\left[\left(\frac{u_e}{u_i}\right)^{-1} - 1\right], \qquad (9)$$

where the constant C will depend on the particle size.

TABLE II

Values of the Ratio, $m\,t_i/C_T$, for Various Runs

Average glass bead size 250 microns; $D/T = 1/3$.

No	C_T		t_i-observed	Slope "m" $C_W t_{100}$ curve	Z,cm	D_e,cm	Z/D_e	$m\,t_i/C_T$
	actual weight percent	apparent volume percent						
38	4·40	7·17	5·45	0·060	1·27	0·476	2·667	0·0456
40	10·00	16·30	5·80	0·137	1·27	0·476	2·667	0·0487
41	10·00	16·30	20·50	0·148	3·81	0·239	15·96	0·186
42	10·00	16·30	16·5	0·095	1·27	0·317	4·0	0·096
6	5·06	8·25	19·0	0·083	3·81	0·239	15·96	0·1911
22	5·00	8·15	21·0	0·035	1·27	0·317	4·00	0·0902
12	5·00	8·15	20·0	0·035	1·27	0·317	4·00	0·086
23	5·00	8·15	1·5	0·046	1·27	0·317	4·00	0·0875
13	5·00	8·15	16·5	0·040	1·27	0·317	4·00	0·081
24	10·00	16·30	22·0	0·052	1·27	0·317	4·00	0·070
54	10·00	16·30	22·5	0·069	1·27	0·317	4·00	0·095
29	10·00	16·30	18·5	0·085	1·27	0·317	4·00	0·0965
53	10·00	16·30	15·5	0·102	1·27	0·317	4·00	0·097
25	10·00	16·30	19·0	0·076	1·27	0·317	4·00	0·088
36	10·00	16·30	31·0	0·106	3·81	0·239	15·96	0·202
37	10·00	16·30	25·0	0·127	3·81	0·239	15·96	0·1948
50	10·00	16·30	10·6	0·070	1·27	0·476	2·667	0·0455
52	10·00	16·30	7·50	0·111	1·27	0·476	2·667	0·051
51	10·00	16·30	6·0	0·160	1·27	0·476	2·667	0·058
16	5·00	8·15	25·0	0·011	1·27	0·317	4·0	0·0338
17	5·00	8·15	20·0	0·018	1·27	0·317	4·0	0·0442
18	10·00	16·30	21·6	0·023	1·27	0·317	4·0	0·0305
19	10·00	16·30	18·5	0·026	1·27	0·317	4·0	0·0295

Finally, it need be mentioned that a few experiments were conducted when the withdrawal tube was not in line with turbine but was positioned 5 cm above and below it. In either case the flow was such that the stream lines passed the opening vertically. When the discharge opening was above the turbine the withdrawn concentration was observed to be always higher than the tank concentration C_T while it was always less than the tank concentration for the other case. The linear nature of the $CW - t_{100}$ curves still prevailed. On the basis of gravitational forces this observation is plausible. Investigations and analysis of such situations is not intended in this work. The only point to be emphasized is that for such cases or when the outlet opening is flush with the tank wall, the radial velocity of approach is zero and the concept of isokinetic withdrawal, analysed hitherto, loses its significance. The difference *in inertia* of the solid particles and their fluid counterparts in affecting the change of direction of the two phases at the outlet opening still play a dominant role.

LIST OF SYMBOLS

A area of opening (cm^2)
B constant for equation *1*
C_W concentration of dispersed phase in the outlet stream
C_T concentration of dispersed phase in the body of the tank
D mixing impeller diameter (m)
D_e diameter of the outlet (m)
H liquid level in tank (m)
$K\,mt/C_T$ constant in Eq. (*5*)
N rotational speed of impeller (rev/s)
T inside diameter of tank
Z insertion length
m constant in Eq. (*4*)
r radial distance from centre of impeller to a position in the tank where velocity is to be determined (m)
t_i time to collect 100 ml of sample at the isokinetic withdrawal
t_{100} time to collect 100 ml of sample
u_e linear velocity of suspension in the outlet (m)
u_i linear velocity of suspension inside the vessel in the vicinity of the outlet at isokinetic withdrawal (m)
u_r centre-line velocity of a stream flowing from a flat-blade mixing turbine (m)
Θ separation coefficient C_W/C_T
ϱ_l liquid density (gm/cm^3)
ϱ_s density of the solid particles (gm/cm^3)

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